SAILING INTO SPACE: STEERING TOWARDS MARS

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TRAVELING THROUGH SPACE

Rockets have limits

- Must carry mass (fuel) to throw away
- Once fuel is gone, have to coast the rest of the way
- Limited energy and little room for improvement
- Can only perform certain missions

A DIFFERENT APPROACH

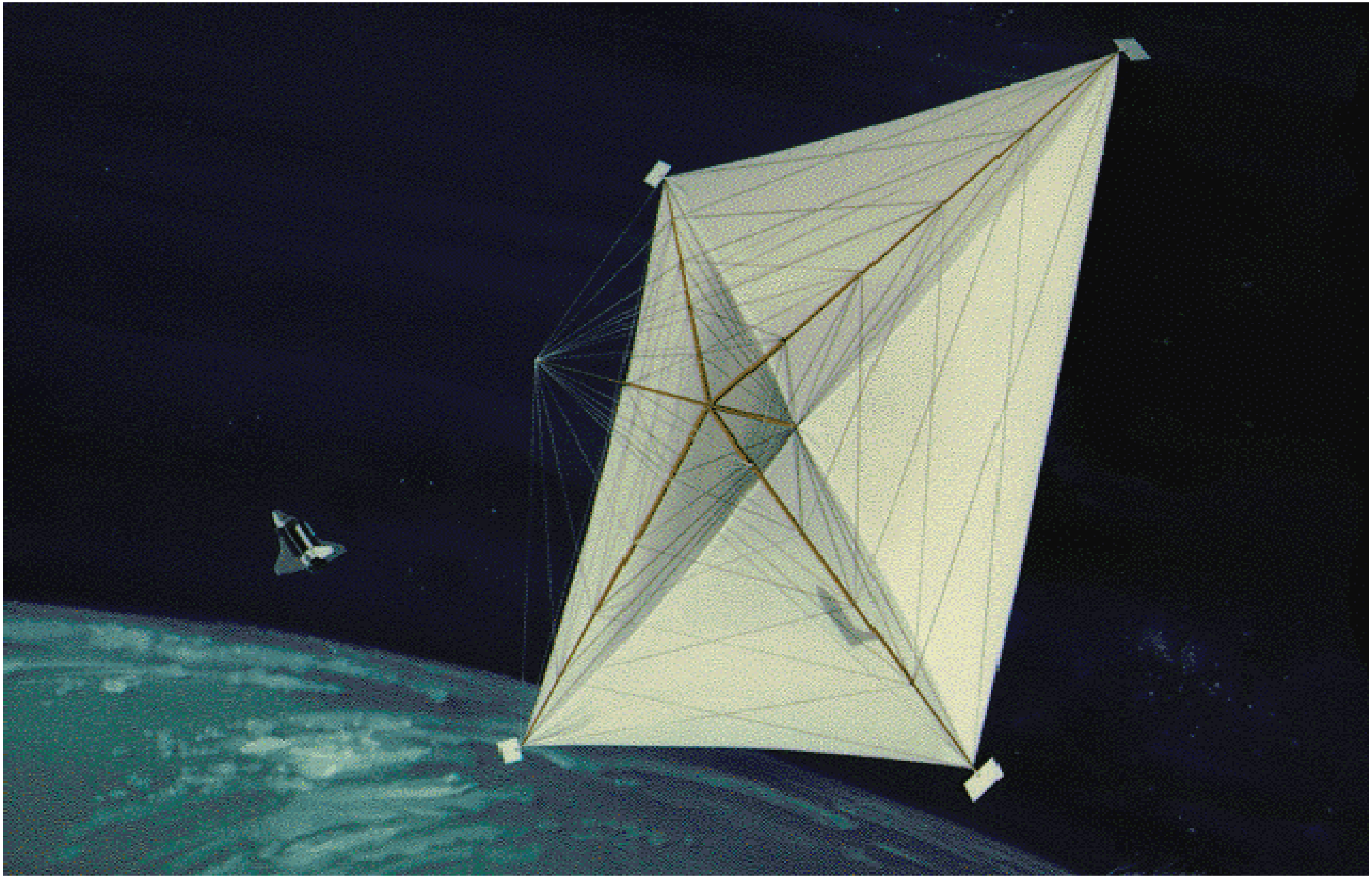
- Needs no fuel
- Uses sunlight (photons)
- Provides continuous acceleration (small, steady increase builds up high velocity over time)
- It's all done with mirrors:

SOLAR SAIL

WHAT MAKES A GOOD SOLAR SAIL?

- High reflectivity (two-for-one push)
  - upon impact
  - upon reflection
- Low mass
  - maximum acceleration (F = m a >>> a = F / m)
- Large area
  - gather a lot of photons (maximize force)
- Combined variable: M/A ratio

SQUARE SOLAR SAIL

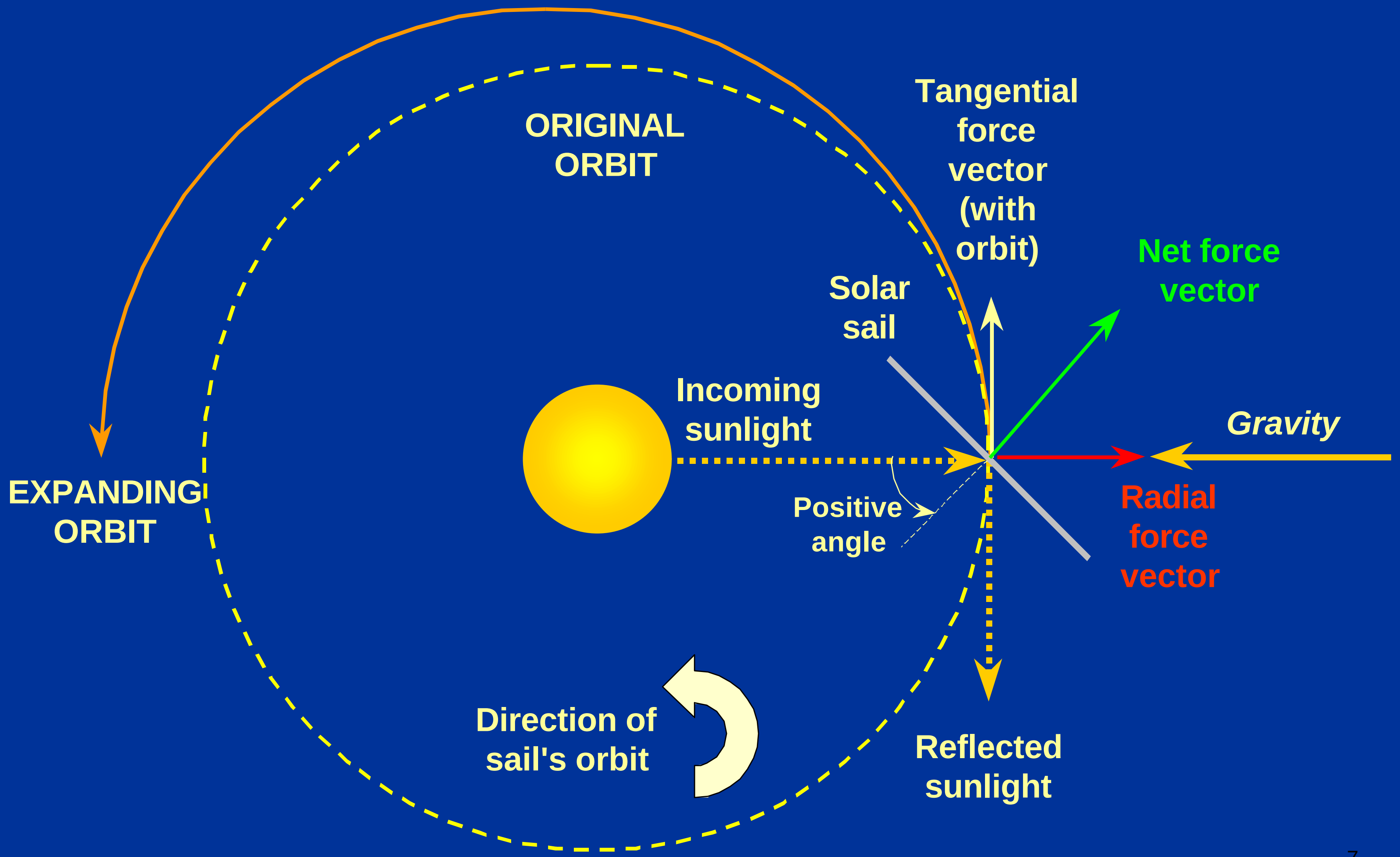


NASA / Jet Propulsion Laboratory

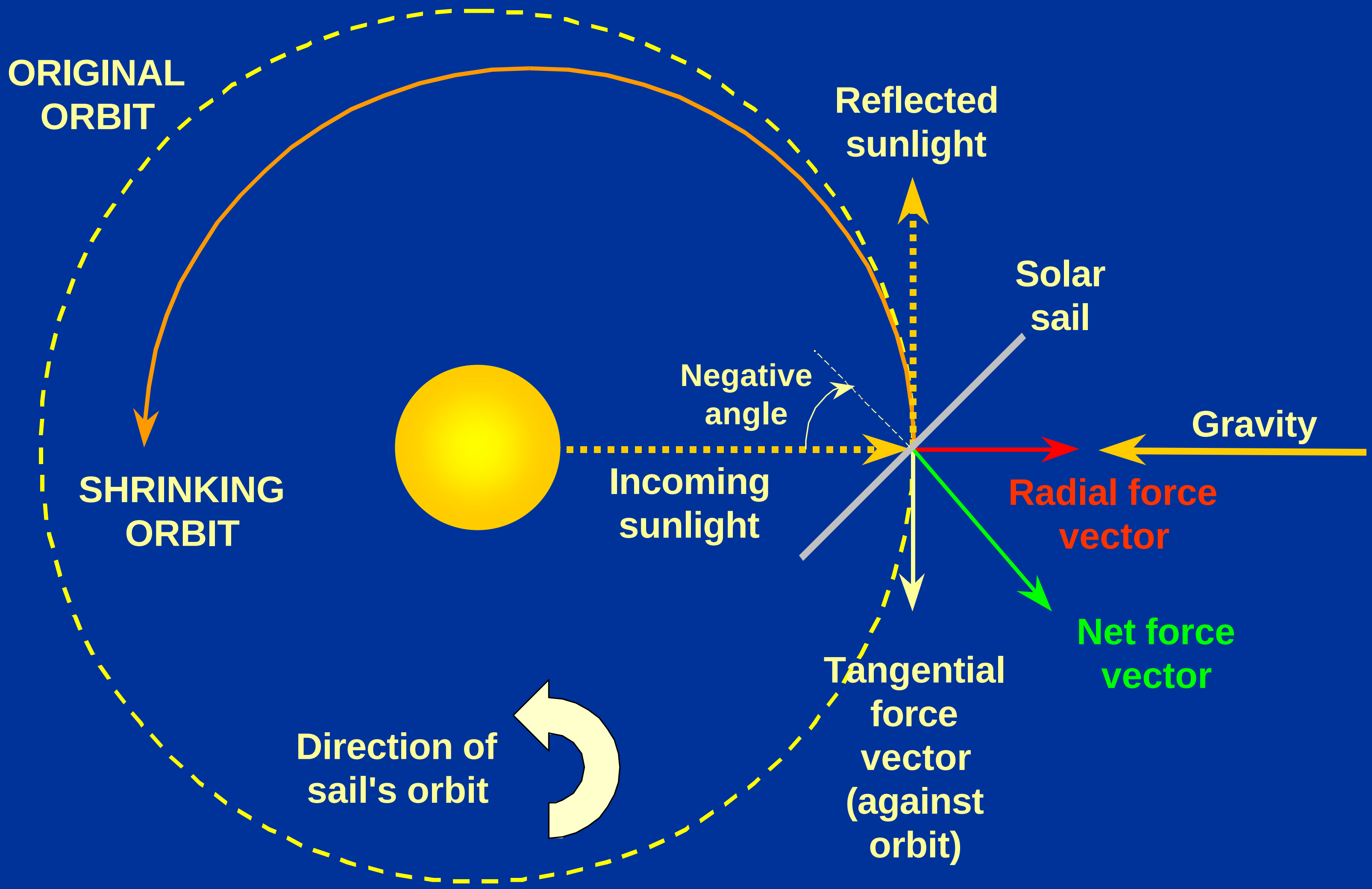
HOW IS IT STEERED?

- By “tacking,” like when sailing on water
- Changing tilt angle of the sail relative to the sunlight
  - positive tilt angle will add energy, sail will spiral out into space
  - negative tilt angle will subtract energy, sail will spiral in towards the sun

FORCE ON A SOLAR SAIL DUE TO SUNLIGHT
(Positive Angle)



FORCE ON A SOLAR SAIL DUE TO SUNLIGHT
(Negative Angle)



PURPOSE

- Investigate if the “best” tilt angle (from last year’s research) yields the shortest travel time to a successful rendezvous with Mars
- Build upon previous work
  - improve accuracy of calculations
- Try to hit a moving target (Mars) from a moving base (Earth)
  - use actual positions of planets and investigate different departure dates

HYPOTHESIS

- A fixed tilt angle of 35.26° will get a solar sail to Mars in the shortest time
  - This angle was found during last year's research
  - It produces the maximum rate of orbit change

RESEARCH PLAN

- Learn how to plot elliptical orbits and compute planetary positions
- Improve accuracy of solar sail path calculations
- Explore various combinations of variables to find the shortest travel time to Mars
  - tilt angle
  - mass to area ratio
  - departure date

ELLIPTICAL ORBITS

- **Exact solution for TWO bodies**
- **However, all planets interact with the Sun *<u>and</u>* each other**
- **Result: very complicated mathematics**
- **Therefore use approximations**
  - **requires no calculus**
  - **accurate to within one arc-minute (1/60th of a degree)**

CALCULATING ELLIPTICAL ORBITS

- **Need to know “orbital elements”**
  - **seven (7) variables that define a planet’s position on a particular calendar date of the current year**
  - **published annually in the <u>Astronomical Almanac</u>**
- **Need to know number of days since orbital elements date**
  - **use Julian date (number of days since January 1, 4713 BC)**
  - **takes into account various adjustments (Julian to Gregorian change, leap years)**

PLANET POSITION

- **Mean longitude**
  - **location of planet in its orbit on date of orbital elements (September 13, 2000)**
- **Average daily motion**
  - **how far planet moves along in its orbit in one (mean solar) day**
- **Mean anomaly**
  - **where planet would be on desired date if orbit was circular**

PLANET POSITION

- **True anomaly**
  - **adjust for elliptical orbit to get planet's actual position (since daily motion varies with position)**
- **Calculate radius vector**
  - **from true anomaly and other properties of ellipses**
- **Calculate coordinates**
  - **from true anomaly, radius vector, and other orbital elements**

SOLAR SAIL POSITION

- ♦ **Improve Euler's method (use Huen's method)**
  - **estimate position from acceleration, step by step**
  - **use "second order" factors (parabolic segments instead of straight lines between steps)**
- ♦ **Depart from Earth's orbital position on a specific date**
  - **use calculated radius vector, and tangential and radial orbital speeds for initial conditions**

PROCEDURE

- Design a solar sail
  - area, material, payload, tilt angle
- Choose a departure date, set mission duration
- Track position relative to Mars
  - if solar sail arrives ahead of Mars, back up departure date
  - if solar sail arrives behind Mars, advance departure date
- Check orbital energy upon reaching Mars

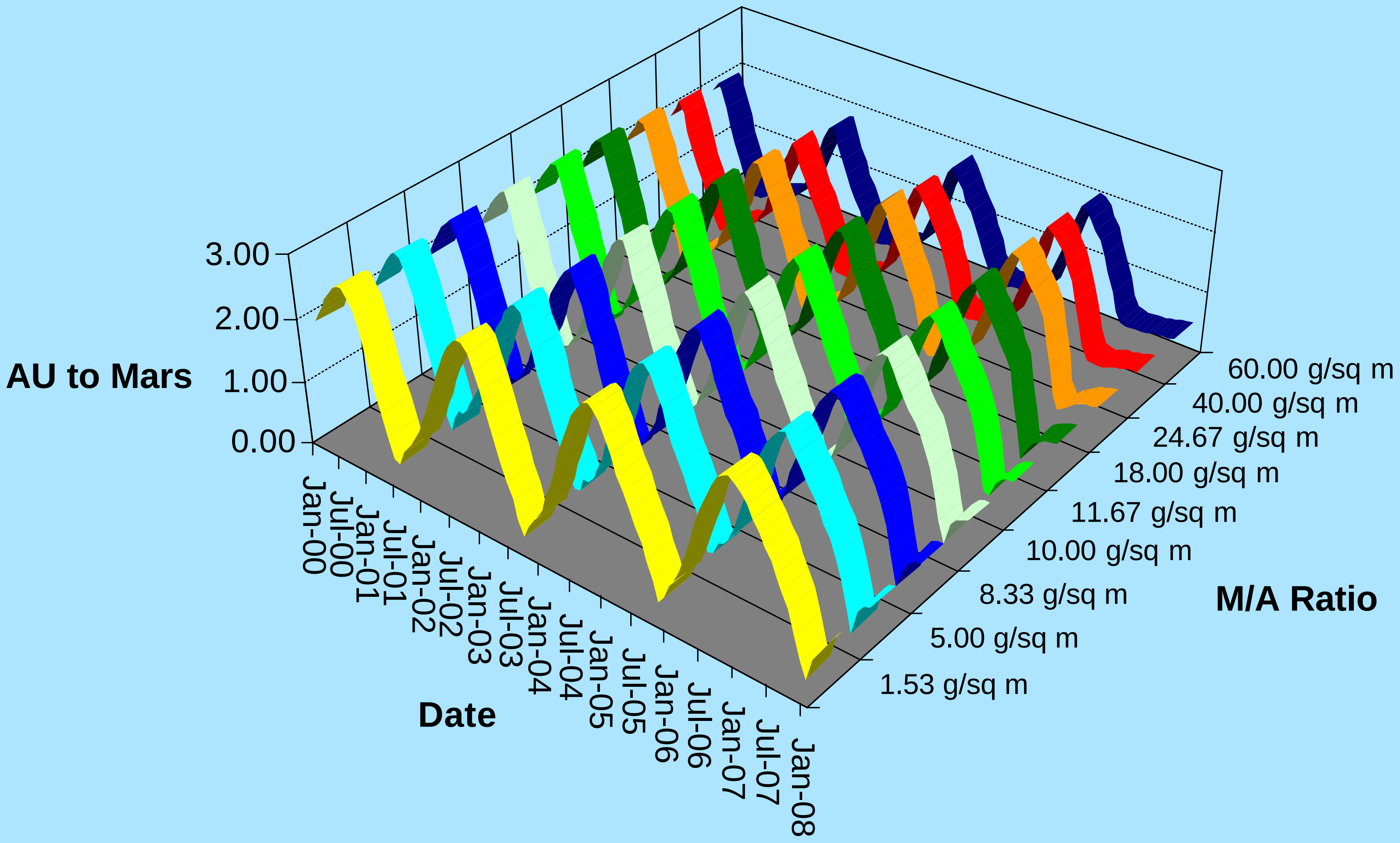
WHAT MAKES THE MISSION A SUCCESS?

- Get to about the same point in space at the same time as the planet Mars (inside the sphere of influence - SOI, radius of about 577,000 km)
- Velocity and distance are “just right” for gravity of Mars to capture the sail
  - » total orbital energy (kinetic - velocity & potential - distance) must be negative

kinetic µ relative velocity^2
(match speeds)

potential µ 1 / relative distance
(get close)

Departure dates (fixed tilt angle = 35.26^o)



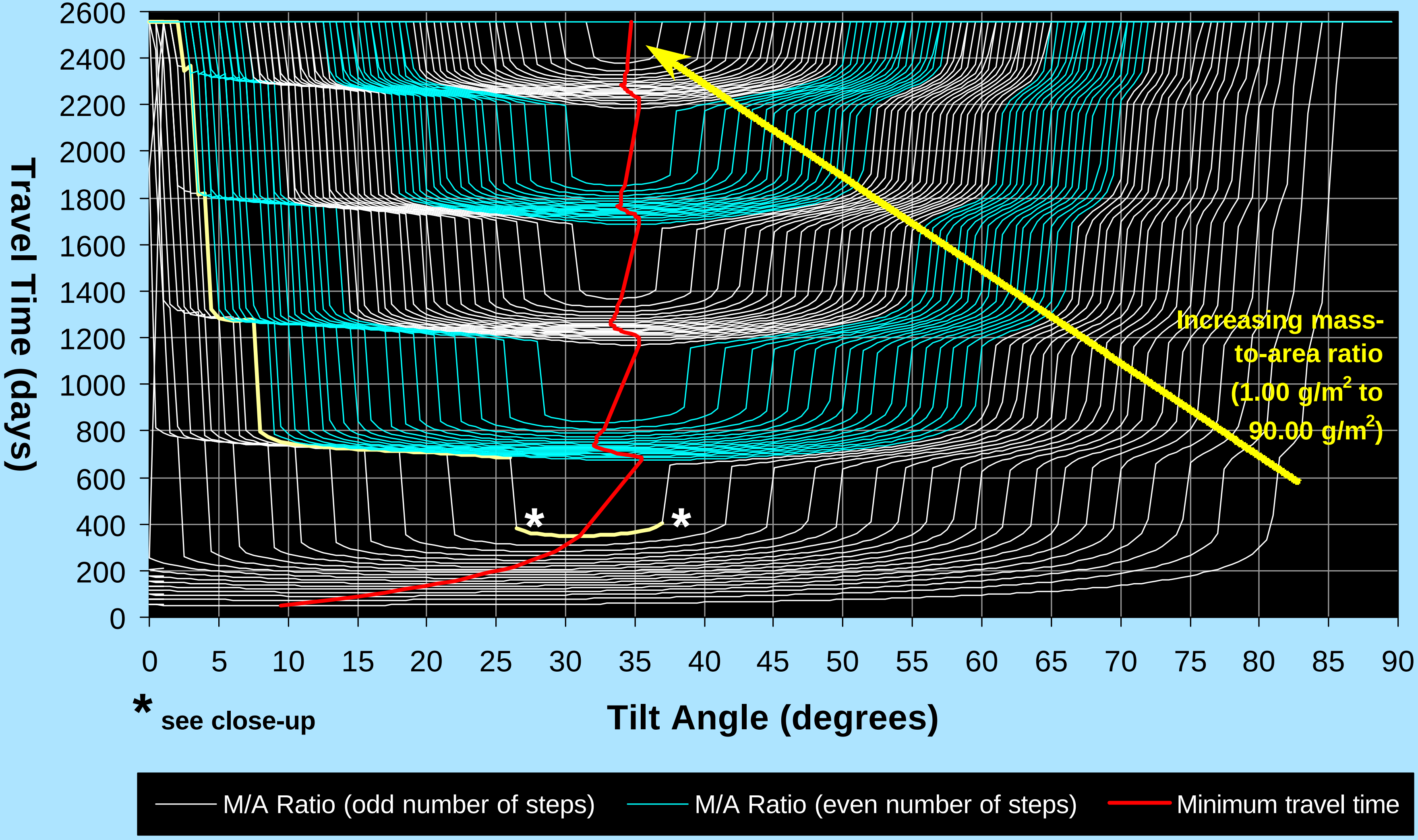
DEPARTURE DATE

- So many to choose from!
- How to match date, tilt angle, and mass-to-area ratio?
  - Calculate travel time from Earth orbit to Mars orbit
  - Create a chart and look for patterns (expected smooth, U-shaped curves with well-defined minimums)

UNEXPECTED RESULTS

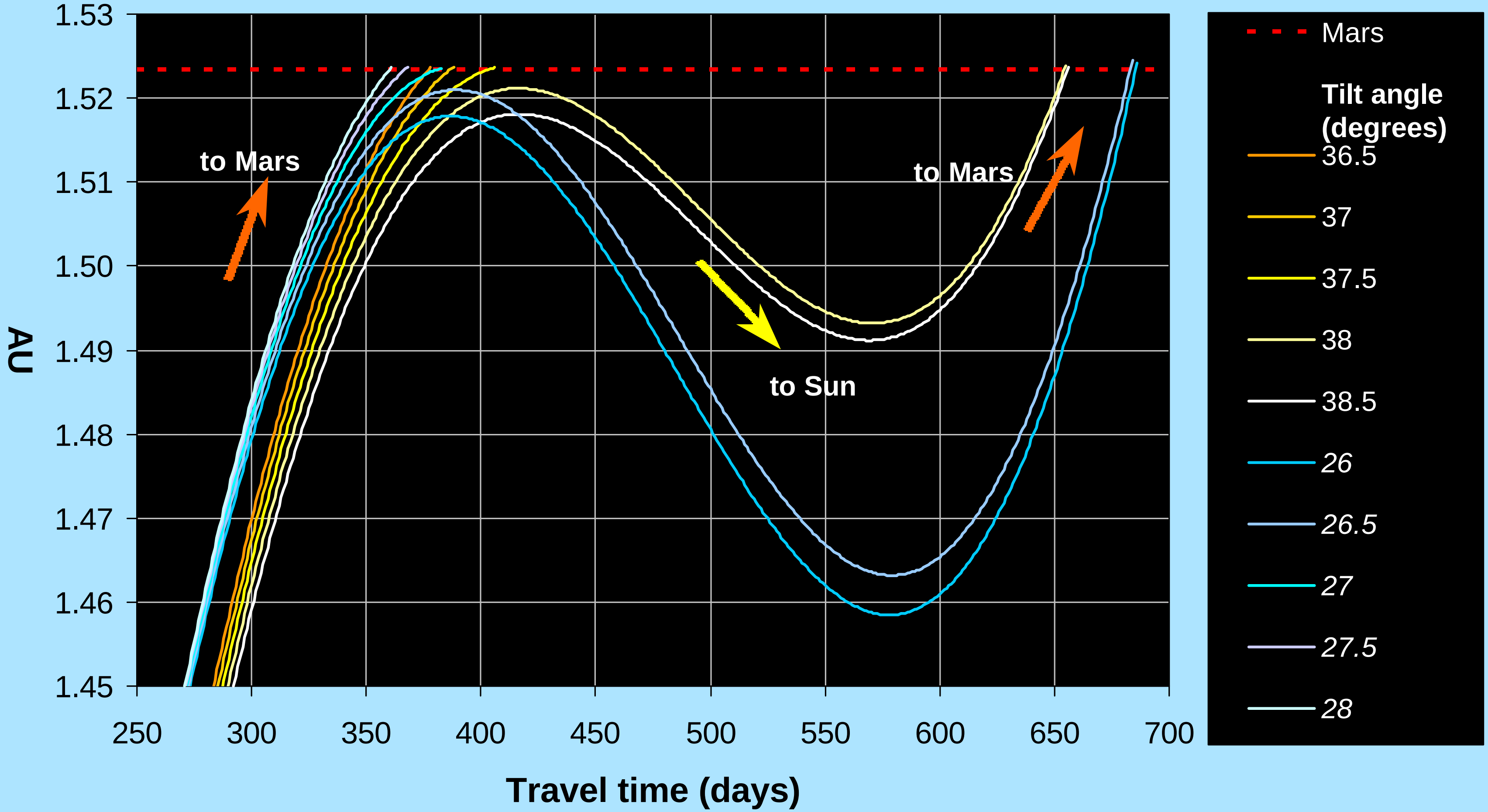
- **“Jumps” of several hundred days in travel time with very small changes in tilt angle**
- **Minimum travel time not always at 35.26° (red line)**
- **Curves have “flat bottoms”**
- **Mass-to-area ratios show groupings (white and blue)**

Time to reach Mars orbit
various M/A ratios and tilt angles

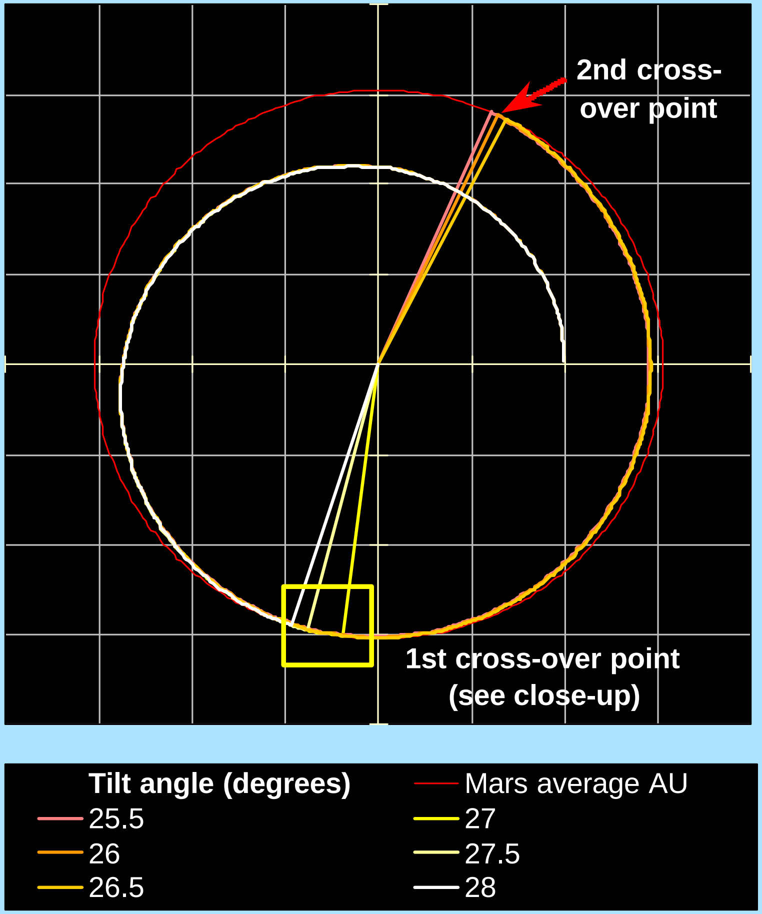


* see close-up

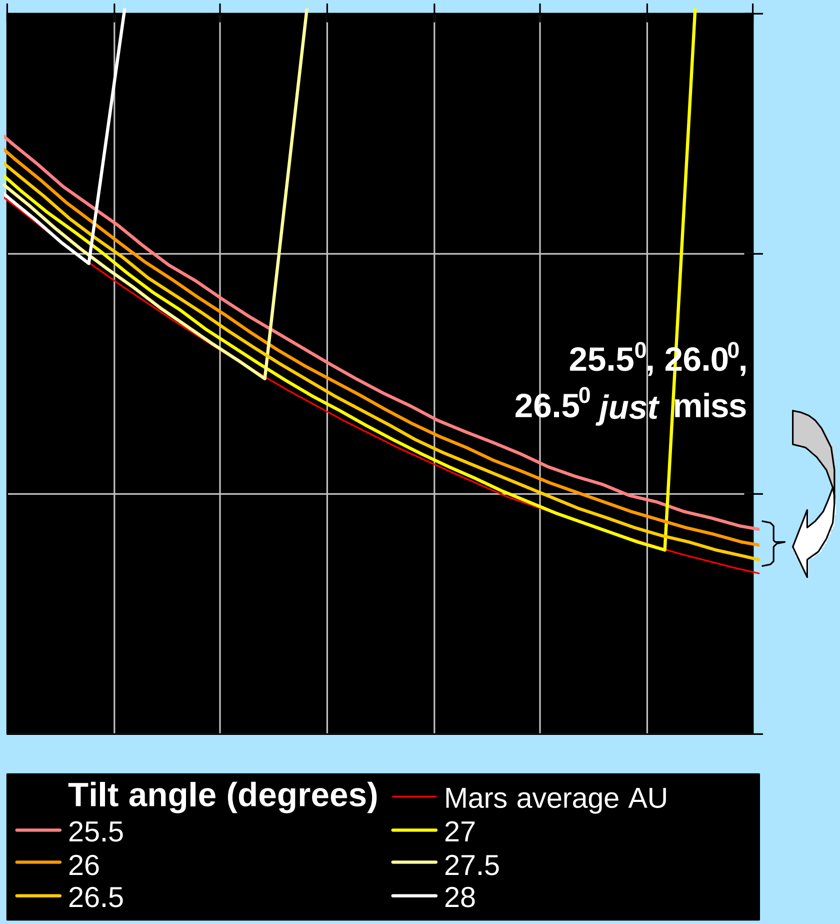
Radial distance from the sun, 19.00 M/A ratio



Orbital cross-overs (19.00 M/A ratio)

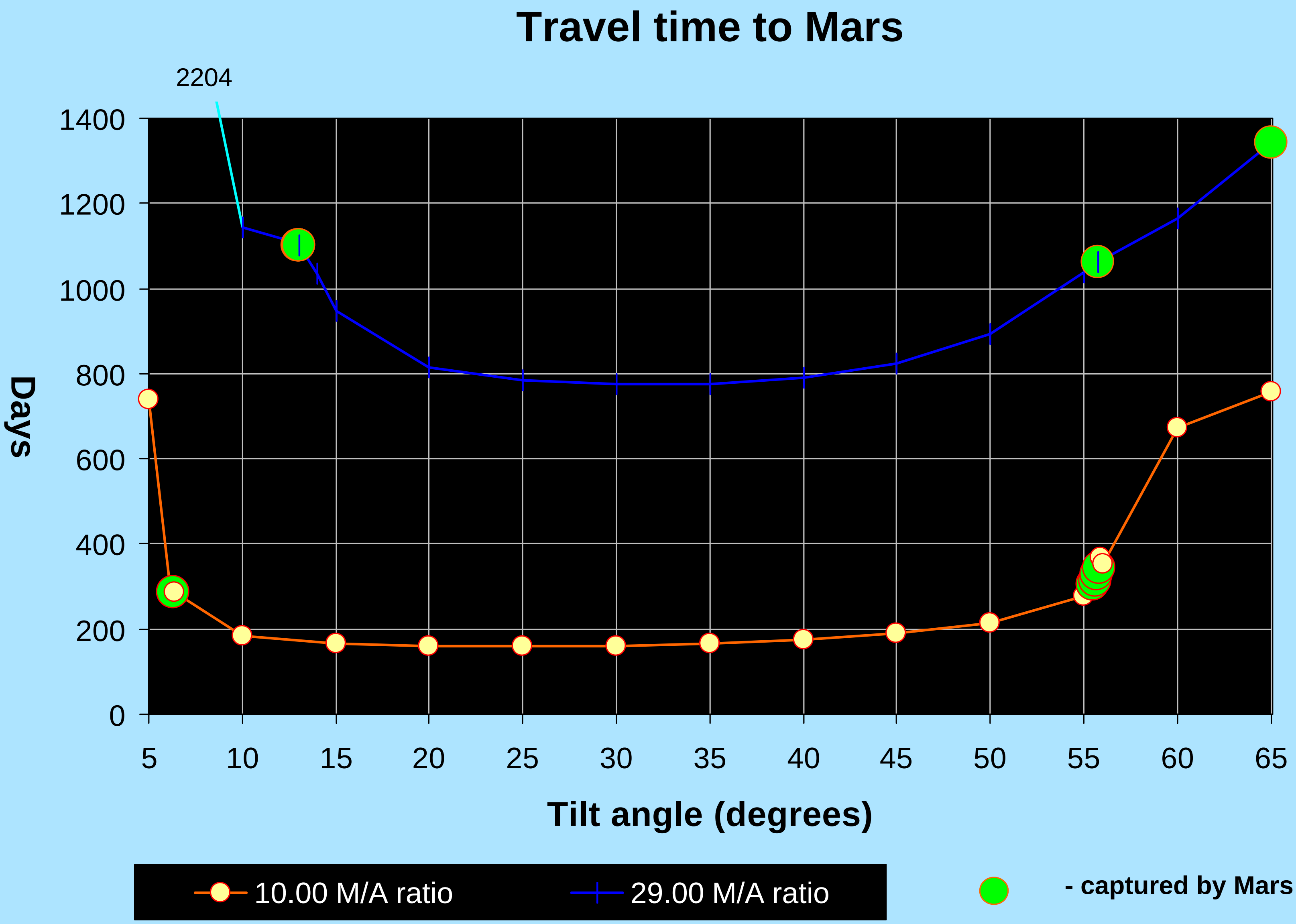


Close-up of first cross-over (vertical scale exaggerated)



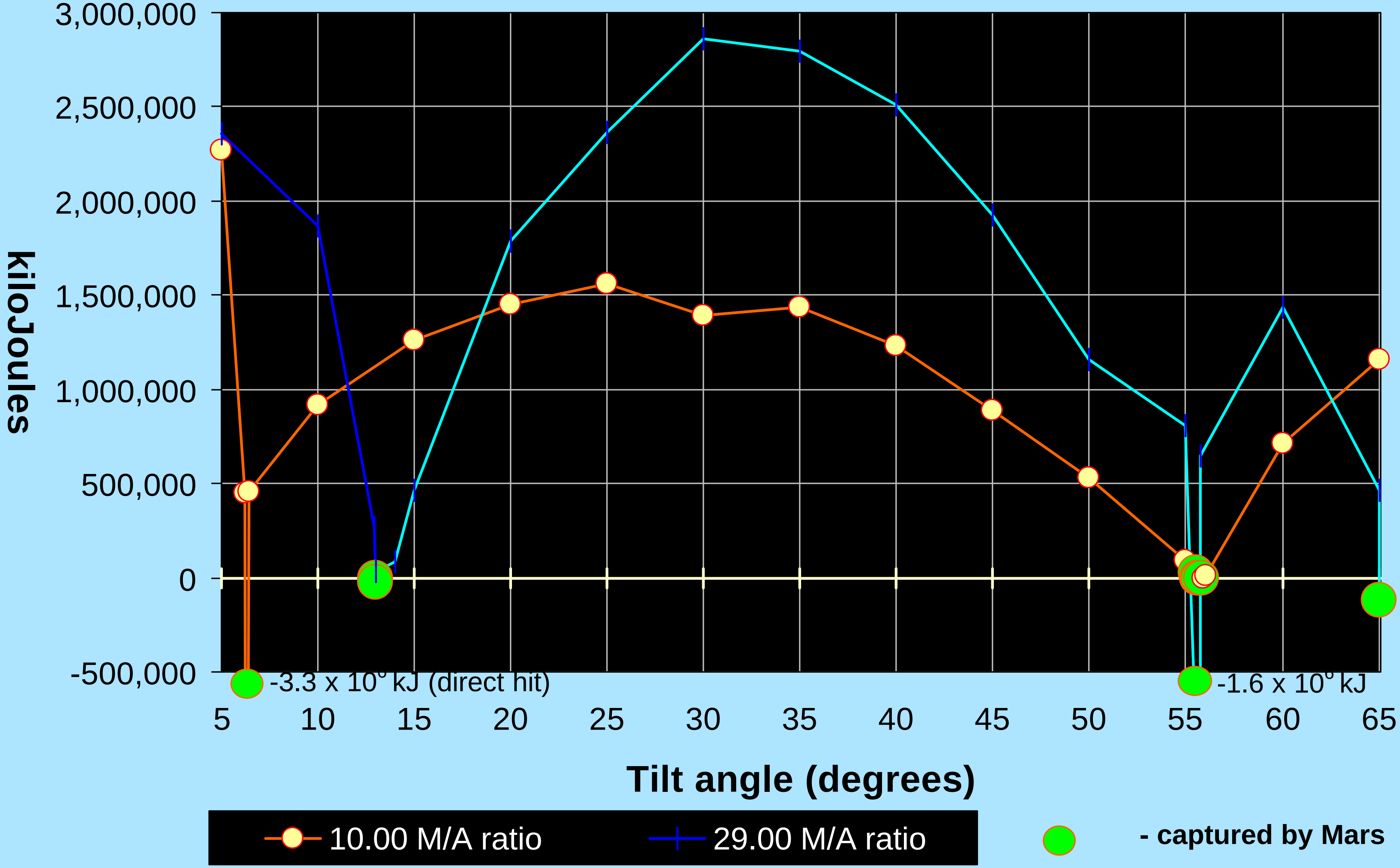
TESTING THE HYPOTHESIS

- ♦ Select representative M/A ratios
    - 10 grams / m^2
      (middle of first group)
    - 29 grams / m^2
      (middle of second group)
- ♦ Compare travel time and orbital energy



Travel time to Mars
2204
1400
1200
1000
800
600
400
200
0
Days
5
10
15
20
25
30
35
40
45
50
55
60
65
Tilt angle (degrees)
10.00 M/A ratio
29.00 M/A ratio
- captured by Mars

Orbital energy at closest approach to Mars



SUCCESSFUL MISSIONS TO MARS

M/A ratio	Tilt angle (degrees)	Depart date	Time to SOI[1] (days)	Arrival date at Mars	Total time (days)	Distance to Mars[2]	Orbital energy (kJ)
10.00	6.3	7-04-03	283.79	4-15-04	286.13	0.17	-3.3×10^6
	55.6	4-23-03	309.29	3-03-04	315.00	11.49	-1,036
	55.7	4-23-03	321.71	3-16-04	328.33	15.86	-582
	55.8	4-23-03	337.63	4-01-04	344.58	24.75	-7
	55.9	4-23-03	360.75	4-24-04	367.13	50.73	-11
29.00	13.001	7-13-02	1,097.71	7-18-05	1,101.79	6.01	-8,217
	13.002	7-13-02	1,097.42	7-18-05	1,101.46	5.84	-20,244
	13.003	7-13-02	1,097.17	7-18-05	1,101.17	6.14	-4,825
	13.004	7-13-02	1,096.88	7-18-05	1,101.00	5.79	-22,481
	55.7484	5-09-02	1,059.17	4-04-05	1,061.67	0.74	-1.6×10^6
	65.0075	9-01-02	1,340.13	5-15-05	1,343.38	2.94	-122,735

[1] Sphere of Influence

[2] Planetary diameter (6800 km)

RESULTS

- Departure date is critical
  - even a one day difference results in a failed mission
- Success is very sensitive to small changes in tilt angle
- Successful missions used either low or high tilt angles
  - need to approach Mars along an almost tangent path

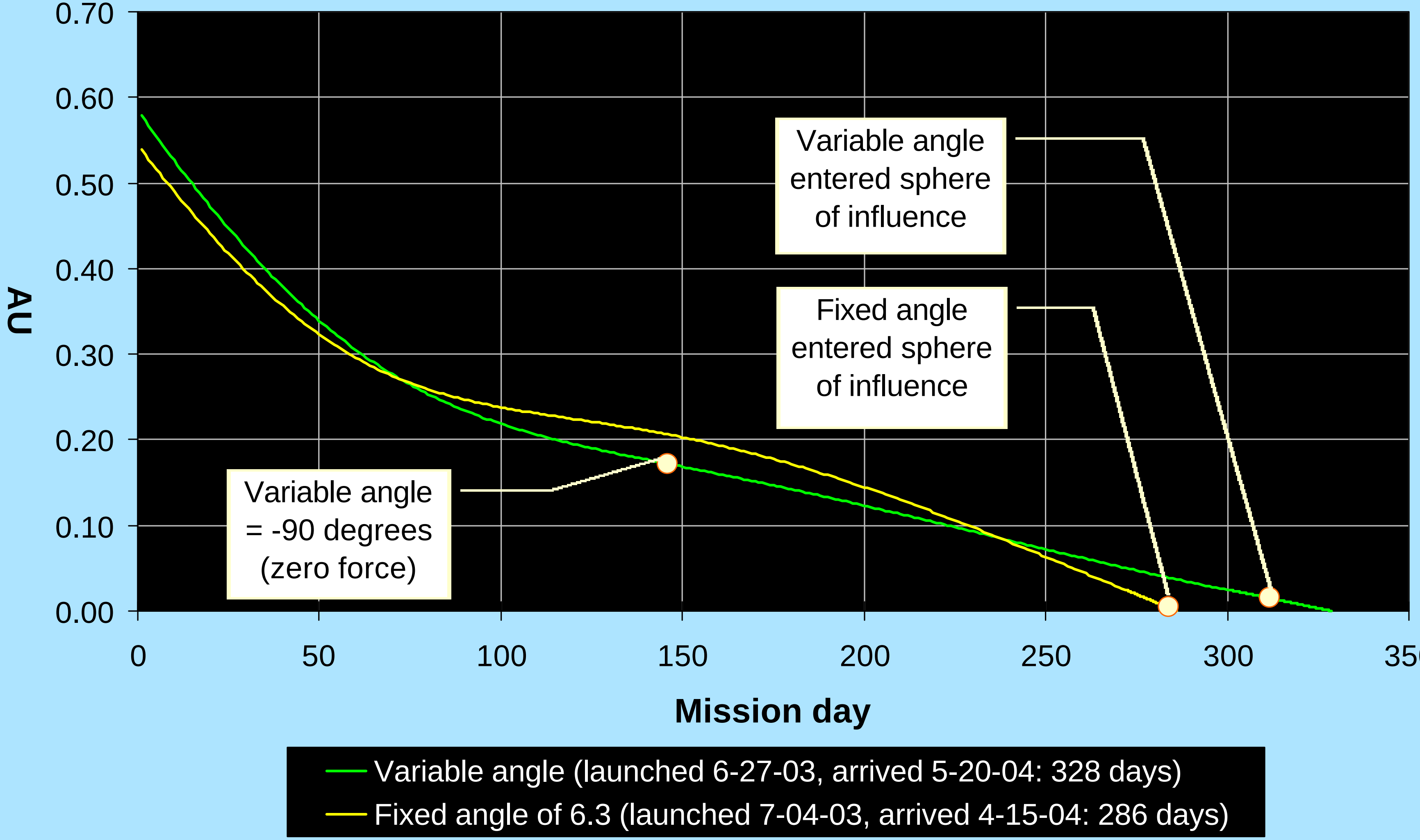
CONCLUSIONS

- The angle that produces the maximum orbit change (35.26°) gives the sail too much energy to rendezvous with Mars
- Small values (6° - 13°) and large values (55° - 65°) produced successful missions
- With the proper M/A ratio and tilt angle, a solar sail can reach Mars from any departure date

FUTURE RESEARCH

- Develop a way to find the parameters for successful missions more efficiently
- Check if continuously adjusting the tilt angle during a mission will get a solar sail to Mars more quickly

Relative distance to Mars, fixed vs. variable angle



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